applies. The voltages and the currents are assumed to satisfy the following equations:

$$V'(x) = -Z(x)I(x) \tag{1}$$

$$I'(x) = -Y(x)V(x). \tag{2}$$

These equations can be written in a matrix form, as in eq. (3) of the paper in question, the solution to which is claimed to be

$$W(x) = \exp\left\{\int_{x_0}^{x} A(t)\,dt\right\} W_0 \tag{3}$$

where $W(x)$ is a column vector with components $V(x)$ and $I(x)$, and $A(x)$ is a 2×2 matrix. It should be pointed out at this point that this is true only if the matrix satisfies the following condition: the matrices $A(x_1)$ and $A(x_2)$ must commute for all x_1 and x_2. To see this, we expand the exponential in its power series (since this is how a function of an operator is defined) to obtain

$$\exp\int_{x_0}^{x} A(t)\,dt = \sum_{n=0}^{\infty} \frac{1}{n!}\left\{\int_{x_0}^{x} A(t)\,dt\right\}^n. \tag{4}$$

Differentiating this series with respect to x (up to second term for clarity), we get

$$A(x) + \frac{1}{2!}\left\{A(x)\int A(t)\,dt + \int A(t)\,dt A(x)\right\} + \cdots. \tag{5}$$

It is clear that (5) reduces to (3) only if the matrix $A(x)$ satisfies the following condition:

$$A(x)A(y) - A(y)A(x) = 0 \tag{6}$$

or, equivalently, that the ratio $Z(x)/Y(x)$ be a constant independent of the position x. Furthermore, the generalization of the stated theorem as presented by the author applies only to matrices satisfying this condition. A constant matrix obviously satisfies condition (6). For matrices which do not satisfy this condition, the transition from eq. (7) to eqs. (9) and (10) in Nwoke's paper cannot be made. To see this, we perform a similarity transformation which diagonalizes the matrix $A(x)$. Of course this similarity transformation is dependent on x since the eigenvectors of the matrix $A(x)$ are position-dependent. Multiplying both sides of Nwoke's eq. (7) by $P(x)^{-1}$ from the left and $P(x)$ from the right, we get

$$P(x)^{-1}\exp\left\{\int_{x_0}^{x} A(t)\,dt\right\}P(x) = \alpha + \beta(\lambda(x)). \tag{7}$$

In this equation $P(x)$ is a 2×2 matrix whose columns are the components of eigenvectors of $A(x)$, and $\lambda(x)$ is a diagonal 2×2 matrix whose diagonal elements are the eigenvalues of $A(x)$. In order for the left-hand side of this equation to be equal to that in eqs. (9) and (10) in the paper in question, the matrix $P(x)$ must diagonalize the matrix $A(y)$ for arbitrary values of x and y. This can be seen by expanding the exponential in its power series and trying to reduce each term in the series to a diagonal form. Thus we obtain the following condition on $Z(x)$ and $Y(x)$.

$$Z(x)/Y(x) = \text{constant} = Z(x_0)/Y(x_0). \tag{8}$$

Finally, note that eqs. (14) and (15) do not satisfy eqs. (1) and (2) in Nwoke's paper unless this condition is met. The appendix carries out this verification but this condition was implicitly assumed in getting this result. Also the solutions to the given examples do not satisfy eqs. (1) and (2), as can be seen by direct differentiation. For lines satisfying this condition, the method is an elegant one. The solution to the general case where $Z(x)$ and $Y(x)$ are not related is very complex and involves an infinite ordered series in the matrix $A(x)$. To be explicit, the general solution to eqs. (1) and (2) in Nwoke's paper can be written formally as

$$(x) = X\exp\int_{x_0}^{x} A(t)\,dt \tag{9}$$

where the operator X orders a product $A(x_1)A(x_2)\cdots A(x_n)$, the arguments x_i appearing in ascending order from right to left. For a more detailed discussion standard books on quantum field theory or quantum many-body theory may be consulted, for example [1]. A perturbation expansion may be fruitful for problems where the series (which is known in quantum field theory as the Dyson series) converges.

References

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Comments on "TE and TM Modes of Some Triangular Cross-Section Waveguides Using Superposition of Plane Waves"

Jingjun Zhang and Junmei Fu

In the above paper,[1] Overfelt and White found the exact transverse electric and magnetic mode solution of four triangular cross-section waveguides: 1) equilateral; 2) 30°, 30°, 120°; 3) isosceles right; and 4) 30°, 60° right triangular. But the work of Prof. Lin Weigan some years ago [1] should not be neglected. His results for 30°, 60° right triangular waveguides are as follows.

With the coordinate system in Fig. 1 with a 30° angle at the origin, for TE modes,

$$H_z = \cos\frac{l\pi x}{a}\cos\frac{(m-n)\pi y}{\sqrt{3}a} + \cos\frac{m\pi x}{a}\cos\frac{(n-l)\pi y}{\sqrt{3}a} + \cos\frac{n\pi x}{a}\cos\frac{(l-m)\pi y}{\sqrt{3}a}, \qquad l+m+n=0.$$

The cutoff wavenumbers for the TE modes are

$$k_c = \frac{2\pi}{\sqrt{3}a}\sqrt{m^2+mn+n^2}$$

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[1] P. L. Overfelt and D. J. White, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 161–167, Jan. 1986.

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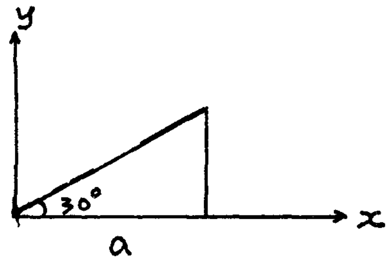

Fig. 1. Cross section of 30°, 60° right triangular waveguide.

where only one of m, n, and l can be zero. That is, the first mode is TE_{01} or TE_{10}.

For TM modes,

$$E_z = \sin\frac{l\pi x}{a}\sin\frac{(m-n)\pi y}{\sqrt{3}\,a} + \sin\frac{m\pi x}{a}\sin\frac{(n-l)\pi y}{\sqrt{3}\,a} + \sin\frac{n\pi x}{a}\sin\frac{(l-m)\pi y}{\sqrt{3}\,a}, \qquad l+m+n=0.$$

The expression for cutoff wavenumbers of TM modes is the same as that for TE modes, but $m \neq 0$, $n \neq 0$, and $m \neq n$; that is, the first mode is TM_{12} or TM_{21}.

Reply[2] *by P. Overfelt and D. J. White*[3]

We appreciate having the paper by Prof. Lin Weigan called to our attention. It is always good to find an interest in a paper one has written, particularly on a subject which many might consider somewhat arcane. In fact, we would appreciate a copy of Prof. Lin's paper to add to our slim file on exact and near-exact solutions for waveguides of polygonal cross section. Our interest in the subject continues.

Our failure to cite this paper lay in our lack of knowledge of its existence. Given the enormous body of scientific and engineering literature in many languages, the problem of properly acknowledging previous work is one that we all have.

The solutions given for the modes in the 30°–60° right triangle are, in fact, identical to ours. Setting Lin's mode indices in capital letters and ours in lowercase and making the substitutions

$$M = (m+n)/2 \qquad N = (m-n)/2$$

or

$$m = M + N \qquad n = M - N$$

it can be seen that the expressions for E_z, H_z, and k_c are identical.

It must be said that his mode index scheme is probably superior to ours (although ours arose naturally out of the manner in which we solved the problem) since it does not have the auxiliary conditions that $m+n$ and $m-n$ be even and is more in line with conventional practice.

[2]Manuscript received October 1, 1990.
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[1] Lin, Weigan, *Microwave Theory and Techniques.* Beijing: Science Press, 1979, pp. 154–162 (in Chinese).